

CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total grade—50

Part A consists of 28 questions that will be answered on side 1 of the answer sheet. Following are the directions for Section I, Part A)

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$.

- (A) -72
- (B) -32
- (C) -24
- (D) 24
- (E) 32

2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

- (A) $x < -2$ or $x > 2$
 - (B) $x \leq -2$ or $x \geq 2$
 - (C) $-2 < x < 2$
 - (D) $-2 \leq x \leq 2$
 - (E) $x \leq 2$
-

GO ON TO THE NEXT PAGE 

3. $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$ is

- (A) 0
 - (B) 10
 - (C) -10
 - (D) 5
 - (E) The limit does not exist
-

4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

- (A) $\frac{(5x^4 - 1)}{(3x^2)}$
 - (B) $\frac{(5x^4 - 1) - (3x^2)}{(x^3 + 7)}$
 - (C) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)}$
 - (D) $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$
 - (E) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$
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GO ON TO THE NEXT PAGE 

5. Evaluate $\lim_{h \rightarrow 0} \frac{5\left(\frac{1}{2} + h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

- (A) $\frac{5}{2}$
- (B) $\frac{5}{16}$
- (C) 40
- (D) 160
- (E) The limit does not exist.

6. $\int x\sqrt{3x} \, dx =$

- (A) $\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$
- (B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$
- (C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$
- (D) $2\sqrt{3x} + C$
- (E) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

GO ON TO THE NEXT PAGE 

7. Find k so that $f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & ; x \neq 4 \\ k & ; x = 4 \end{cases}$ is continuous for all x .

- (A) All real values of k make $f(x)$ continuous for all x .
 - (B) 0
 - (C) 16
 - (D) 8
 - (E) There is no real value of k that makes $f(x)$ continuous for all x .
-

8. Which of the following integrals correctly gives the area of the region consisting of all points above the x -axis and below the curve $y = 8 + 2x - x^2$?

- (A) $\int_{-2}^4 (x^2 - 2x - 8) dx$
 - (B) $\int_{-4}^2 (8 + 2x - x^2) dx$
 - (C) $\int_{-2}^4 (8 + 2x - x^2) dx$
 - (D) $\int_{-4}^2 (x^2 - 2x - 8) dx$
 - (E) $\int_2^4 (8 + 2x - x^2) dx$
-

9. If $f(x) = x^2 \cos 2x$, find $f'(x)$.

- (A) $2x \sin 2x$
 - (B) $-2x \cos 2x + 2x^2 \sin 2x$
 - (C) $-4x \sin 2x$
 - (D) $2x \cos 2x - 2x^2 \sin 2x$
 - (E) $2x - 2 \sin 2x$
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GO ON TO THE NEXT PAGE 

10. An equation of the line tangent to $y = 4x^3 - 7x^2$ at $x = 3$ is

- (A) $y + 45 = 66(x + 3)$
 - (B) $y - 45 = 66(x - 3)$
 - (C) $y = 66x$
 - (D) $y = 66(x - 3)$
 - (E) $y - 45 = \frac{-1}{66}(x - 3)$
-

11. $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}} dx =$

- (A) $\frac{\pi}{6}$
 - (B) $\frac{\pi}{3}$
 - (C) $-\frac{\pi}{3}$
 - (D) $\frac{2\pi}{3}$
 - (E) $-\frac{2\pi}{3}$
-

12. Find a positive value c , for x , that satisfies the conclusion of the Mean Value Theorem for Derivatives for $f(x) = 3x^2 - 5x + 1$ on the interval $[2, 5]$.

- (A) 1
 - (B) $\frac{13}{6}$
 - (C) $\frac{11}{6}$
 - (D) $\frac{23}{6}$
 - (E) $\frac{7}{2}$
-


GO ON TO THE NEXT PAGE 

13. Given $f(x) = 2x^2 - 7x - 10$, find the absolute maximum of $f(x)$, on $[-1, 3]$.

- (A) -1
 - (B) $\frac{7}{4}$
 - (C) -13
 - (D) $-\frac{129}{8}$
 - (E) 0
-

14. Find $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$.

- (A) $(3x^2 + 3xy^2)$
 - (B) $-(3x^2 + 3xy^2)$
 - (C) $\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$
 - (D) $-\frac{(3x^2y + y^3)}{(3xy^2 + x^3)}$
 - (E) $-\frac{(x^2y + y^3)}{(xy^2 + x^3)}$
-

GO ON TO THE NEXT PAGE 

15. If $f(x) = \sqrt{1 + \sqrt{x}}$, find $f'(x)$.

(A) $\frac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\frac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\frac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

16. $\int 7xe^{3x^2} dx =$

(A) $\frac{1}{42}e^{3x^2} + C$

(B) $\frac{6}{7}e^{3x^2} + C$

(C) $\frac{7}{6}e^{3x^2} + C$

(D) $7e^{3x^2} + C$

(E) $42e^{3x^2} + C$

GO ON TO THE NEXT PAGE 

17. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through $(2, -1)$.

- (A) $-9x + 8y - 26 = 0$
 - (B) $9x - 8y - 26 = 0$
 - (C) $9x - 8y - 106 = 0$
 - (D) $8x + 9y - 17 = 0$
 - (E) $9x + 16y - 2 = 0$
-

18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is its acceleration at time $t = 4$?

- (A) 0
 - (B) 9
 - (C) -9
 - (D) -12
 - (E) 12
-

19. If $f(x) = 3^{\pi x}$, then $f'(x) =$

- (A) $\frac{3^{\pi x}}{\pi \ln 3}$
 - (B) $\frac{3^{\pi x}}{\ln 3}$
 - (C) $\frac{3^{\pi x}}{\pi}$
 - (D) $\pi \cdot 3^{\pi x - 1}$
 - (E) $\pi \ln 3 \cdot 3^{\pi x}$
-

GO ON TO THE NEXT PAGE 

20. The average value of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = e$ is

(A) $\frac{1}{e+1}$

(B) $\frac{1}{1-e}$

(C) $e-1$

(D) $1 - \frac{1}{e^2}$

(E) $\frac{1}{e-1}$

21. If $f(x) = \sin^2 x$, find $f'''(x)$.

(A) $-\sin^2 x$

(B) $2\cos 2x$

(C) $\cos 2x$

(D) $-4\sin 2x$

(E) $-\sin 2x$

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22. Find the slope of the normal line to $y = x + \cos xy$ at $(0,1)$.

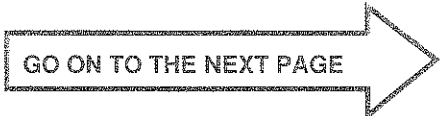
- (A) 1
 - (B) -1
 - (C) 0
 - (D) 2
 - (E) Undefined
-

23. $\int e^x \cdot e^{3x} dx =$

- (A) $\frac{1}{3}e^{3x} + C$
 - (B) $\frac{1}{4}e^{4x} + C$
 - (C) $\frac{1}{4}e^{5x} + C$
 - (D) $4e^{4x} + C$
 - (E) $4e^{5x} + C$
-

24. $\lim_{x \rightarrow 0} \frac{\tan^3(2x)}{x^3} =$

- (A) -8
 - (B) -2
 - (C) 2
 - (D) 8
 - (E) The limit does not exist.
-

GO ON TO THE NEXT PAGE 

25. A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x -axis, and the y -axis is revolved about the x -axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi \int_0^1 (1 + \sin^4 x) dx$

(B) $\pi \int_0^1 (1 + \sin^2 x)^2 dx$

(C) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^4 x) dx$

(D) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x)^2 dx$

(E) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x) dx$

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26. If $y = \left(\frac{x^3 - 2}{2x^5 - 1} \right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

(A) -52

(B) -28

(C) -13

(D) 13

(E) 52

GO ON TO THE NEXT PAGE 

27. $\int x\sqrt{5-x} \, dx =$

(A) $-\frac{10}{3}(5-x)^{\frac{3}{2}}$

(B) $\sqrt{\frac{5x^2}{2} - \frac{x^3}{3}} + C$

(C) $\frac{10}{3}\sqrt{\frac{5x^2}{2} - \frac{x^3}{3}} + C$

(D) $10(5-x)^{\frac{1}{2}} + \frac{2}{3}(5-x)^{\frac{3}{2}} + C$

(E) $-\frac{10}{3}(5-x)^{\frac{3}{2}} + \frac{2}{5}(5-x)^{\frac{5}{2}} + C$

28. If $\frac{dy}{dx} = \frac{x^3+1}{y}$ and $y=2$ when $x=1$, then, when $x=2$, $y=$

(A) $\sqrt{\frac{27}{2}}$

(B) $\sqrt{\frac{27}{8}}$

(C) $\pm\sqrt{\frac{27}{8}}$

(D) $\pm\frac{3}{2}$

(E) $\pm\sqrt{\frac{27}{2}}$

GO ON TO THE NEXT PAGE 

Part B consists of 17 questions that will be answered on side 2 of the answer sheet. Following are the directions for Section I, Part B)

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING SIDE 2 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 29–45.

YOU MAY NOT RETURN TO SIDE 1 OF THE ANSWER SHEET.

In this test:

1. The *exact* numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
 2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
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29. The graph of $y = 5x^4 - x^5$ has an inflection point (or points) at
- (A) $x = 0$ only
 - (B) $x = 3$ only
 - (C) $x = 0, 3$
 - (D) $x = -3$ only
 - (E) $x = 0, -3$
-
30. The average value of $f(x) = e^{4x^2}$ on the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$ is
- (A) 0.272
 - (B) 0.545
 - (C) 1.090
 - (D) 2.180
 - (E) 4.360
-

GO ON TO THE NEXT PAGE 

31. $\int_0^1 \tan x \, dx =$

- (A) 0
 - (B) $\frac{\tan^2 1}{2}$
 - (C) $\ln(\cos(1))$
 - (D) $\ln(\sec(1))$
 - (E) $\ln(\sec(1)) - 1$
-

32. $\frac{d}{dx} \int_0^{x^2} \sin^2 t \, dt =$

- (A) $x^2 \sin^2(x^2)$
 - (B) $2x \sin^2(x^2)$
 - (C) $\sin^2(x^2)$
 - (D) $x^2 \cos^2(x^2)$
 - (E) $2x \cos^2(x^2)$
-

GO ON TO THE NEXT PAGE 

33. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

- (A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm\frac{2}{3}$ (E) $\pm\frac{3}{2}$
-

34. The graph of $y = x^3 - 2x^2 - 5x + 2$ has a local maximum at

- (A) $(2.120, 0)$
(B) $(2.120, -8.061)$
(C) $(-0.786, 0)$
(D) $(-0.786, 4.209)$
(E) $(0.666, -1.926)$
-

35. Approximate $\int_0^1 \sin^2 x \, dx$ using the trapezoid rule with $n = 4$, to three decimal places.

- (A) 0.277
(B) 0.273
(C) 0.555
(D) 1.109
(E) 2.219
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GO ON TO THE NEXT PAGE 

36. The volume generated by revolving about the x -axis the region above the curve $y = x^3$, below the line $y = 1$, and between $x = 0$ and $x = 1$ is

- (A) $\frac{\pi}{42}$
(B) 0.143π
(C) $\frac{\pi}{7}$
(D) 0.643π
(E) $\frac{6\pi}{7}$
-

37. A 20 foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor? (in ft/sec).

- (A) 0.346 (B) 2.887 (C) 0.224 (D) 5.774 (E) 4.472
-

38. $\int \frac{\ln x}{3x} dx =$

- (A) $6\ln^2|x| + C$
(B) $\frac{1}{6}\ln(\ln|x|) + C$
(C) $\frac{1}{3}\ln^2|x| + C$
(D) $\frac{1}{6}\ln^2|x| + C$
(E) $\frac{1}{3}\ln|x| + C$
-

GO ON TO THE NEXT PAGE 

39. Find two nonnegative numbers x and y whose sum is 100 and for which xy^2 is a maximum.

- (A) $x = 33.333$ and $y = 33.333$
 - (B) $x = 50$ and $y = 50$
 - (C) $x = 33.333$ and $y = 66.667$
 - (D) $x = 100$ and $y = 0$
 - (E) $x = 66.667$ and $y = 33.333$
-

40. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

- (A) 6.000
 - (B) 1.609
 - (C) 16.047
 - (D) 0.800
 - (E) 148.413
-

GO ON TO THE NEXT PAGE 

41. $\int \sin^5(2x)\cos(2x) dx =$

(A) $\frac{\sin^6 2x}{12} + C$

(B) $\frac{\sin^6 2x}{6} + C$

(C) $\frac{\sin^6 2x}{3} + C$

(D) $\frac{\cos^5 2x}{3} + C$

(E) $\frac{\cos^5 2x}{6} + C$

42. The volume of a cube is increasing at a rate proportional to its volume at any time t . If the volume is 8 ft³ originally, and 12 ft³ after 5 seconds, what is its volume at $t = 12$ seconds?

(A) 21.169

(B) 22.941

(C) 16.000

(D) 28.800

(E) 17.600

43. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$.

(A) 0.068

(B) 1.350

(C) 5.400

(D) 6.750

(E) 540.000

GO ON TO THE NEXT PAGE 

44. A particle's height at a time $t \geq 0$ is given by $h(t) = 100t - 16t^2$. What is its maximum height?
- (A) 312.500 (B) 156.250 (C) 78.125 (D) 6.250 (E) 3.125
-

45. If $f(x)$ is continuous and differentiable and $f(x) = \begin{cases} ax^4 + 5x & ; x \leq 2 \\ bx^2 - 3x & ; x > 2 \end{cases}$, then $b =$
- (A) 0.5
(B) 0
(C) 2
(D) 6
(E) There is no value of B
-

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR NUMBER CORRECTLY IN SECTION C OF THE ANSWER SHEET

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three decimal places.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

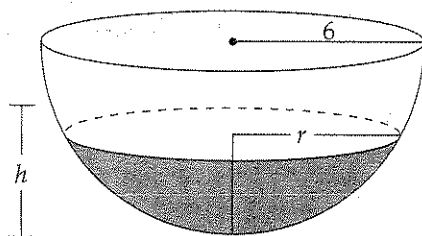
1. Consider the curve defined by $y = x^4 + 4x^3$.
 - (A) Find the equation of the tangent line to the curve at $x = -1$.
 - (B) Find the coordinates of the absolute minimum.
 - (C) Find the coordinates of the point(s) of inflection.
-
2. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight ($0 \leq H < 24$).
 - (A) The initial temperature at midnight was $-15^\circ F$, and at Noon of New Year's Day was $5^\circ F$. Find A and B .
 - (B) Find the average temperature for the first 10 hours.
 - (C) Use the Trapezoidal Rule with 4 equal subdivisions to estimate $\int_0^8 T(H) dH$.
 - (D) Find an expression for the rate that the temperature is changing with respect to H .
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GO ON TO THE NEXT PAGE 

3. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.
- (A) Find an expression for G , the amount of grass in the lake (in tons), in terms of t , the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.
- (B) In how many years will the amount of grass available be 300 tons?
- (C) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

SECTION B

4. Water is being poured into a hemispherical bowl of radius 6 inches at the rate of $4 \text{ in}^3/\text{sec}$.



- (A) Given that the volume of the water in the spherical segment shown above is $V = \pi h^2 \left(R - \frac{h}{3} \right)$, where R is the radius of the sphere, find the rate that the water level is rising when the water is 2 inches deep.
- (B) Find an expression for r , the radius of the surface of the spherical segment of water, in terms of h .
- (C) How fast is the circular area of the surface of the spherical segment of water growing (in in^2/sec) when the water is 2 inches deep?
5. Let R be the region in the first quadrant bounded by $y^2 = x$ and $x^2 = y$.
- (A) Find the area of region R .
- (B) Find the volume of the solid generated when R is revolved about the x -axis.
- (C) The section of a certain solid cut by any plane perpendicular to the x -axis is a semicircle with the ends of its side lying on the parabolas $y^2 = x$ and $x^2 = y$. Find the volume of the solid.

GO ON TO THE NEXT PAGE

6. An object moves with velocity $v(t) = t^2 - 8t + 7$.

- (A) Write a polynomial expression for the position of the particle at any time $t \geq 0$.
 - (B) At what time(s) is the particle changing direction?
 - (C) Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.
-

END OF EXAMINATION